UNIVERSITY OF CAMBRIDGE

MPhil in Scientific Computing

MHD Simulations of Magnetic Reconnection

Motivation

Magnetic reconnection is a complex physical process involving **rapid changes in magnetic field topology** thought to be responsible for energetic outbursts in astrophysical and laboratory plasmas.

This work implements **Resistive and Hall MHD** models, primarily using simulations based on the **GEM challenge** [1], to examine fundamental properties of reconnecting plasmas and the **impact of resistivity and two-fluid effects on reconnection rates and global plasma dynamics.**

Methods

Formulation: Resistive MHD + Hall effect

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot \left(\rho \boldsymbol{v} \otimes \boldsymbol{v} + (p + \frac{1}{2}B^2 \boldsymbol{I} - \boldsymbol{B} \otimes \boldsymbol{B} \right) = 0$$

$$\frac{\partial E}{\partial t} + \nabla \cdot \left(\left(E + p + \frac{1}{2}B^2 \right) - (\boldsymbol{v} \cdot \boldsymbol{B})\boldsymbol{B} \right) = -\eta \nabla \cdot (\boldsymbol{j} \times \boldsymbol{B})$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \cdot (\boldsymbol{v} \otimes \boldsymbol{B} - \boldsymbol{B} \otimes \boldsymbol{v}) = -\nabla \times (\eta \boldsymbol{j})$$

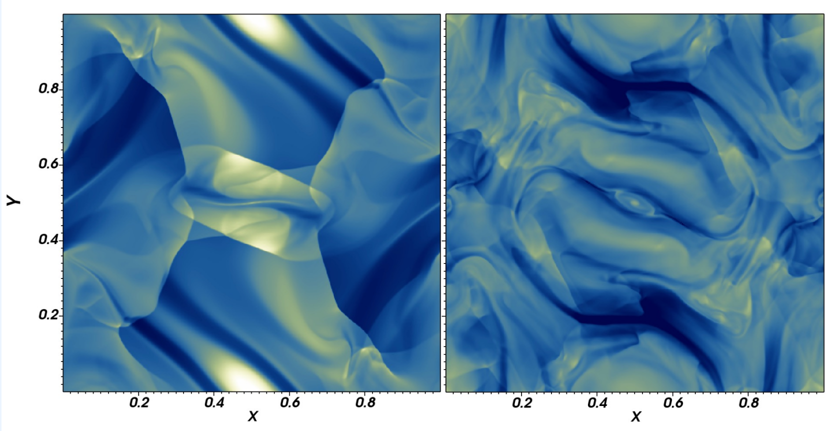
Divergence constraint: $\nabla \cdot (\boldsymbol{B}) = 0$

Ohm's law: $\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \eta \boldsymbol{j} + \frac{1}{ne} \boldsymbol{j} \times \boldsymbol{B}$

Numerical methods: HLLD hyperbolic solver, MUSCL-Hancock with van Leer slope limiter, constrained transport, Athena++ fluid code [2]

Wrote Hall MHD module for Athena++

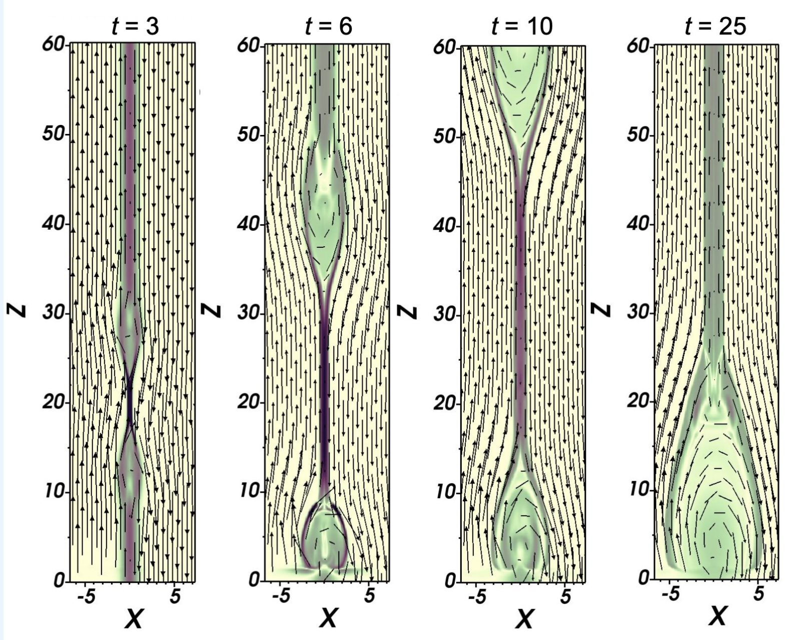
Hall MHD Validation



Orszag-Tang vortex test of supersonic turbulence

Visible dispersion effects from Hall contribution

Figure 1: Density at t=0.5 and t=1.0



Reconnection-driven solar flare

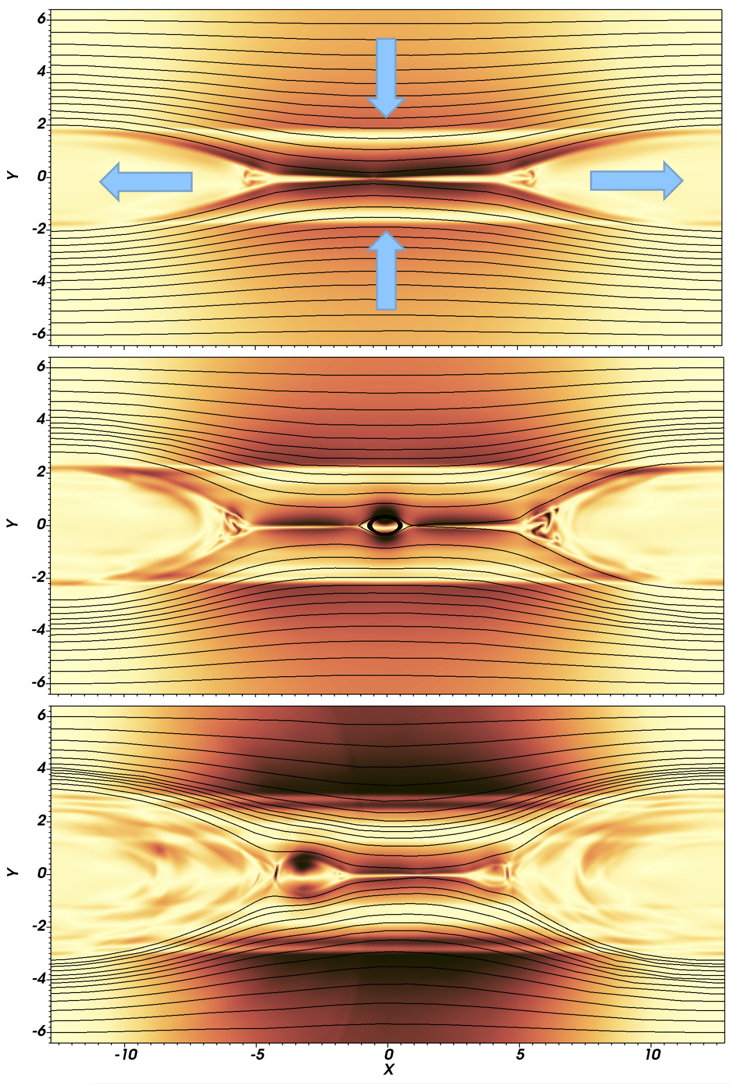
2.5D current sheet setup, density gradient for atmosphere

X-point Hall reconnection leads to ejection of hot plasmoids indicative of coronal mass ejections

Figure 2: Time evolution of current density and magnetic field vectors

Results for GEM Reconnection

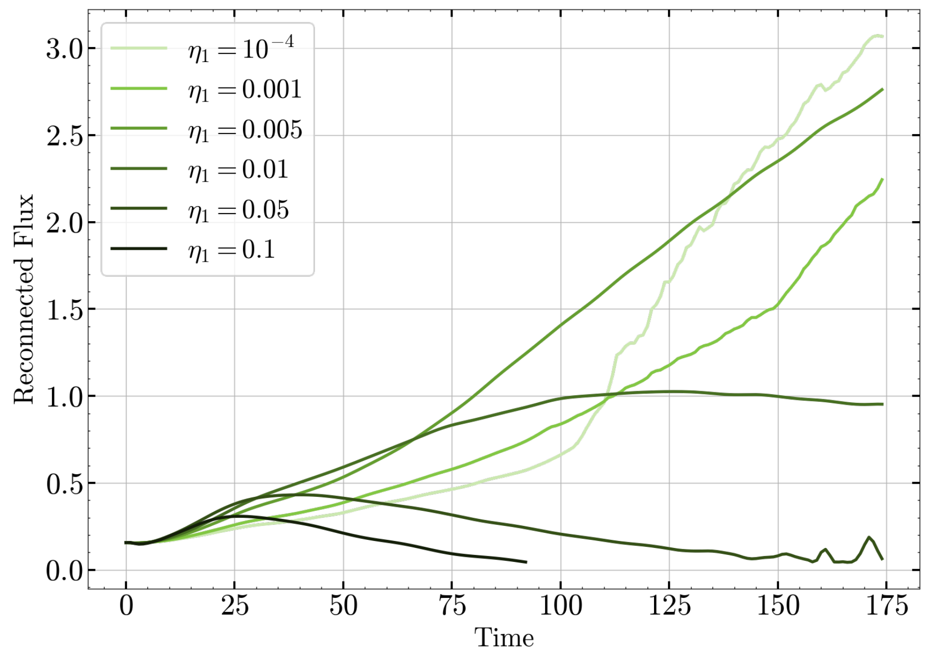
The GEM setup uses a perturbed Harris sheet to trigger a tearing mode instability, leading to fast reconnection.



In low-resistivity plasmas, **magnetic islands** coalesce and are ejected along the current sheet, **destabilizing** previously steady-state reconnection.

This effect is **not observed** in plasmas with spatially localised resistivity profiles or Hall MHD.

Figure 3: current density and magnetic field line evolution for $\eta = 0.001$ in Resistive MHD



For Resistive MHD, reconnected flux **increases** with resistivity **until a critical point** of $\eta = 0.005$, when can no longer support reconnection due to excess diffusion.

Figure 4: Reconnected flux over time for Res. MHD with constant resistivity; line darkness increases with η

Hall MHD reconnection advances 2-3x faster than Resistive MHD **early** on, but the formation of magnetic islands in Res. MHD accelerates reconnected flux later.

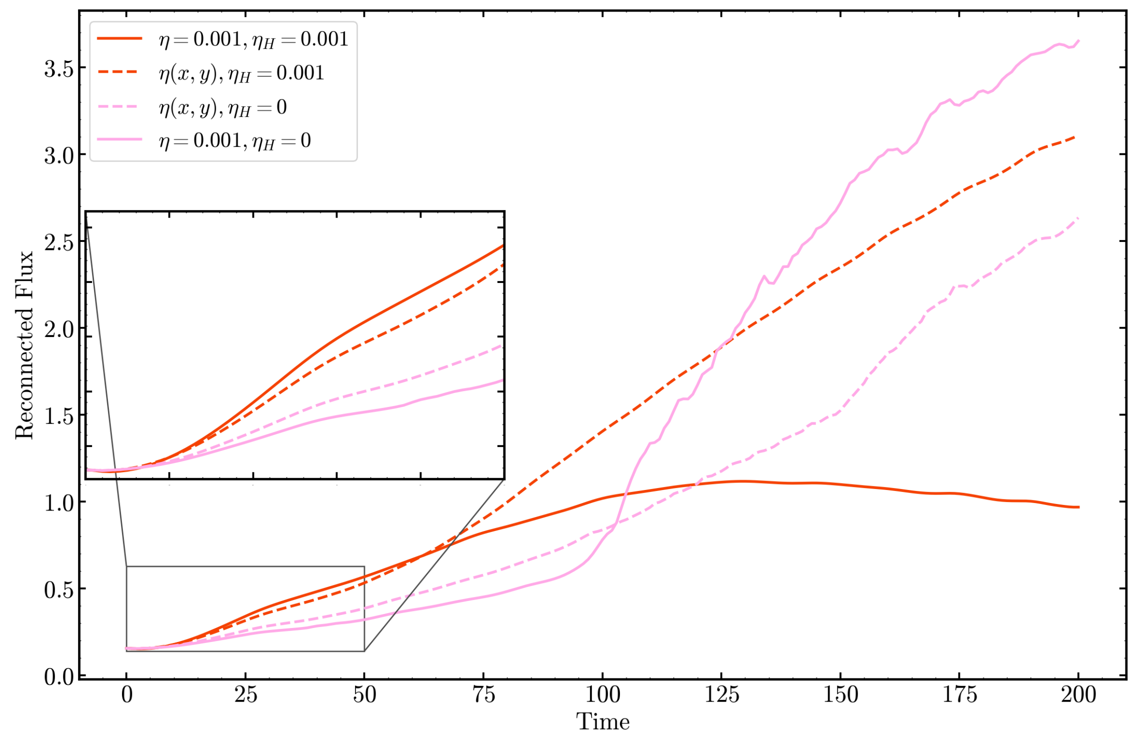


Figure 5: Reconnected flux for Hall (red) vs Resistive (pink)

References and Acknowledgements

[1] J. Birn et al. 2001 — [2] Stone et al. 2020

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